

# Classical and Quantum Coherent State Description of $N\bar{N}$ Annihilation at rest in the Skyrme Model with $\omega$ Mesons

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## Abstract

We model the  $N\bar{N}$  system at rest as a baryon number zero lump in the Skyrme model including the  $\omega$  field. We integrate the classical equations of motion from the highly non-perturbative annihilation region into the non-interacting radiation zone. Subsequent coherent state quantization of the radiation field gives a good description of the pion spectrum from annihilation at rest.

## I. INTRODUCTION

We have recently shown that low energy nucleon-antinucleon annihilation directly into pions can be well described as occurring through a coherent classical pion wave that emerges very quickly from the annihilation region [1], [2]. The physical, quantum pions detected from annihilation can be described by quantizing this classical pion burst using the method of coherent states [3]. We project these coherent states onto states of fixed four-momentum [4] and definite isospin [5]. A correct picture of the pion spectrum, pion number probability and charge averages emerges from this simple picture. Our approach is inspired by calculations of Skyrmion anti-Skyrmion annihilation, where it is found that the classical Skyrmion fields annihilate into a burst of pion radiation nearly as soon as the Skyrmion and anti-Skyrmion touch, and that that pion wave takes away the baryon number and energy as fast as causality will permit [6], [7]. This picture, and our first treatment dealt only with the direct pions and did not address the roughly 40% of annihilations that go first through meson resonances, mostly omega and rho mesons, and then to pions [8]. The purpose of this paper is to begin to address this deficiency by including omega mesons in our description of annihilation. We will return to rho mesons in a subsequent paper.

In our previous work [1], [2], we approached the classical pion wave from annihilation phenomenologically, and concentrated on projections of the coherent state. We found that a very simple parameterization of the wave source gave a good description of the pion spectrum. The only parameter we needed was the overall amplitude of the wave, and this we could fit to the total energy released. When coupling to omega mesons the situation is more complicated, since both the relative amplitude and shape of the omega and pion waves, even classically, should be dynamically determined. Thus we need to make a dynamical theory of the coupled pion and omega fields. Again we do that classically. We construct a classical picture of the annihilation proceeding into interacting pion and omega fields. We propagate those fields, classically, into the radiation zone, where they no longer interact, and then quantize the radiation into coherent states for the pions and omegas separately. Finally we allow the omega mesons to decay into pions, assuming that that radiation is incoherent with respect to the direct pions. This approach of treating the non-perturbative aspects of QCD classically and then quantizing the subsequent radiation in the radiation zone using the methods of coherent states has recently appeared in a number of applications [9], [10].

For a dynamical theory of coupled, classical pion and omega fields, we use the Skyrme model modified to include omegas [11]. Recall that the usual Skyrme lagrangian [12] describes a classical pion field only, and that baryons emerge as topologically stable configurations of that pion field. Also recall that this theory is a candidate for what QCD looks like in the classical or large number of colors limit. To include omega mesons in this formalism, the classical omega field is coupled to the baryon current. This helps to stabilize the Skyrmion. To model annihilation at rest in this formalism, we follow our earlier work [7] and begin with a spherically symmetric “blob” of Skyrmionic matter of size about 1 fm, baryon number zero and the total energy of two nucleons. This initial configuration is not a static solution of the Skyrme equations, but rather evolves according to those equations into radiating and interacting pion and omega fields. As these fields radiate outwards, they diminish like  $1/r$ , decouple and become radiating free fields. We use these radiation fields to construct coherent states for the omegas and pions. We then project these coherent states

onto states of good four-momentum and isospin, as before, and examine the consequences for nucleon-antinucleon phenomenology. We find very good agreement with the principal features of annihilation, now including the omega channel. The coherent state picture seems to work in spite of the fact that the mean number of omegas is of order one. We assume that omega decay occurs outside the interaction region of the classical fields and therefore we do not treat the pions from omega decay as coherent with the direct pions. But we do, of course, treat the pions from the omega as part of the observed pion spectrum.

In Section II we present the classical Skyrme lagrangian with coupling to the omega field and the dynamical equations for propagation of the fields from a spherically symmetric source. We also show the classical field configurations that emerge from the decay of an initial “blob” meant to model annihilation. Section III discusses the quantization of the classical  $\pi$  and  $\omega$  fields using the method of coherent states modified to include projection onto fixed isospin and four-momentum. Section IV gives the results of this formalism for the pion spectrum from annihilation. We find that the mean number of pions, the variance, and the entire shape of the pion number spectrum closely resemble experiment [13]. In our previous, pions only, calculation [2], we found that only an even number of pions could come from even isospin states and only an odd number from odd. The pions from omega decay remove this artifact, giving smooth pion number probabilities in each isospin channel. Section V presents some conclusions and directions for future work.

## II. CLASSICAL CALCULATION

We begin with a classical theory of the coupled pion and omega fields based on the Skyrme model. This coupled theory was written down by Adkins and Nappi [11]. In terms of the usual  $SU(2)$  valued unitary field  $U$  of the Skyrme picture the lagrangian is given by

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_\omega + \mathcal{L}_{int} \quad (1)$$

$$\mathcal{L}_\pi = -\frac{f_\pi^2}{2} \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right) + \frac{1}{2} m_\pi^2 f_\pi^2 \text{Tr} (U - 1) \quad (2)$$

where

$$U = \exp\left(\frac{i}{f_\pi} \vec{\tau} \cdot \vec{\pi}\right) \quad (3)$$

$$\mathcal{L}_\omega = -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{m^2}{2} \omega_\mu \omega^\mu \quad (4)$$

$$\mathcal{L}_{int} = \beta \omega_\mu B^\mu \quad (5)$$

$$B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left[ (U^\dagger \partial_\nu U) (U^\dagger \partial_\alpha U) (U^\dagger \partial_\beta U) \right] \quad (6)$$

The first term,  $\mathcal{L}_\pi$ , is the standard non-linear sigma model term with a pion mass term included. It has two parameters,  $f_\pi$ , the pion decay constant, which it is customary to vary a little to fit the nucleon mass, and the pion mass, which we do not vary. Note that there is no Skyrme term in the lagrangian, since the coupling to the omega now stabilizes the theory. The second term,  $\mathcal{L}_\omega$ , is the free omega lagrangian, where  $\omega_\mu$  is the omega field,  $\omega_{\mu\nu}$  the corresponding antisymmetric field strength tensor, and  $m$  the omega mass. The interaction term,  $\mathcal{L}_{int}$ , couples the omega field to the baryon current,  $B^\mu$ , and  $\beta$  is a coupling parameter. We take  $\beta = 15.6$  and  $f_\pi = 62.0\text{MeV}$  from the work of Adkins and Nappi [11].  $\mathcal{L}_{int}$  describes the coupling of the omega to three pions.

To model annihilation, we will take for our initial configuration a spherically symmetric “blob” of matter with zero baryon number but total energy of two nucleons. Thus we only need solutions of the lagrangian that have spherical symmetry. This is a great simplification. For the spherically symmetric case we may write

$$U = \exp(i\vec{\tau} \cdot \hat{r}F(r, t)) \quad (7)$$

in which case the lagrangian becomes,

$$\begin{aligned} \mathcal{L} = & \frac{f_\pi^2}{2} \left[ \left( \frac{\partial F}{\partial t} \right)^2 - 2 \frac{\sin^2 F}{r^2} - \left( \frac{\partial F}{\partial r} \right)^2 \right] + m_\pi^2 f_\pi^2 (\cos F - 1) \\ & + \frac{1}{2} \left[ \left( \frac{\partial \omega_r}{\partial t} - \frac{\partial \omega_0}{\partial r} \right)^2 + \left( \frac{\partial \omega_\theta}{\partial t} \right)^2 + \left( \frac{\partial \omega_\phi}{\partial t} \right)^2 \right] + \frac{m^2}{2} [\omega_0^2 - \omega_r^2 - \omega_\theta^2 - \omega_\phi^2] \\ & - \frac{\beta}{2\pi^2} \frac{\sin^2 F}{r^2} \left[ \omega_0 \frac{\partial F}{\partial r} - \omega_r \frac{\partial F}{\partial t} \right] \end{aligned} \quad (8)$$

As we can see,  $\omega_\theta$  and  $\omega_\phi$  are completely decoupled from the  $\pi$  field. We therefore shall not consider these two components.

The energy of the system is given by

$$H = 4\pi \int_0^\infty r^2 dr \mathcal{H}(r) \quad (9)$$

where

$$\begin{aligned} \mathcal{H}(r) = & \frac{f_\pi^2}{2} \left[ \left( \frac{\partial F}{\partial t} \right)^2 + \left( \frac{\partial F}{\partial r} \right)^2 + 2 \frac{\sin^2 F}{r^2} \right] + f_\pi^2 m_\pi^2 (1 - \cos F) + \frac{1}{2} \left[ \left( \frac{\partial \omega_r}{\partial t} \right)^2 \right. \\ & \left. - \left( \frac{\partial \omega_0}{\partial r} \right)^2 \right] + \frac{m^2}{2} [\omega_r^2 - \omega_0^2] + \frac{\beta}{2\pi^2} \sin^2 F \left( \omega_0 \frac{\partial F}{\partial r} - \omega_r \frac{\partial F}{\partial t} \right). \end{aligned} \quad (10)$$

The equations of motion are given by

$$f_\pi^2 \left[ \frac{\partial^2 F}{\partial t^2} - \frac{\partial^2 F}{\partial r^2} - \frac{2}{r} \frac{\partial F}{\partial r} + \frac{\sin 2F}{r^2} + m_\pi^2 \sin F \right] = \frac{\beta}{2\pi^2} \frac{\sin^2 F}{r^2} \left( \frac{\partial \omega_0}{\partial r} - \frac{\partial \omega_r}{\partial t} \right), \quad (11)$$

$$\left[ \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + m^2 \right] \omega_0 = \frac{\beta}{2\pi^2} \frac{\sin^2 F}{r^2} \frac{\partial F}{\partial r}, \quad (12)$$

$$\left[ \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + \frac{2}{r^2} + m^2 \right] \omega_r = \frac{\beta}{2\pi^2} \frac{\sin^2 F}{r^2} \frac{\partial F}{\partial t}. \quad (13)$$

We model the initial *static* field configuration as

$$F(r, t = 0) = h \frac{r}{r^2 + a^2} \exp(-r/a) \quad (14)$$

with  $a = 1/m_\pi$  and  $h$  determined by the total energy. This form corresponds to a compact initial “blob” of zero baryon number. Since we want to begin with the system at rest we take

$$\dot{F} = \dot{\omega}_r = \dot{\omega}_0 = 0. \quad (15)$$

We may also take

$$\omega_r(r, t = 0) = 0 \quad (16)$$

since  $B_r = 0$  at  $t = 0$ . These conditions determine  $\omega_0(r, t = 0)$ , we find

$$\omega_0(r, t = 0) = \int_0^\infty dr' G(r, r') \left[ -\beta r'^2 B^0(r') \right] \quad (17)$$

where

$$G(r, r') = \frac{1}{2mrr'} \left( e^{-m|r-r'|} - e^{-m(r+r')} \right) \quad (18)$$

Using the initial values we integrate the equations of motion to determine the fields at later times. As the fields propagate outwards, they diminish in size, making the non-linear and coupling terms in the lagrangian less important. Thus we can define a radiation zone where the fields propagate as linear free fields. In the radiation zone, we can calculate the pion,  $f(k)$ , and omega,  $g(k)$ , momentum distribution amplitudes from the expressions

$$\frac{dN_\pi}{d^3k} = |f(k)|^2 = \frac{1}{\pi k_0^\pi} f_\pi^2 \left| \int_0^\infty dr r^2 j_1(kr) (k_0^\pi + i \frac{\partial}{\partial t}) F(r, t) \right|^2, \quad (19)$$

and

$$\frac{dN_\omega}{d^3k} = |g(k)|^2 = \frac{1}{\pi k_0^\omega} \left| \int_0^\infty dr r^2 j_1(kr) (k_0^\omega + i \frac{\partial}{\partial t}) \omega_r(r, t) \right|^2, \quad (20)$$

respectively.

The results of our calculations are shown in the figures. We have fixed  $h$  in our initial configuration by the requirement that our initial state have total energy of 2 GeV. Figure 1 shows the pion field configuration,  $F$ , as a function of  $r$  and  $t$ , both in Fermi. We see that it emerges as a coherent pulse. Much of that pulse travels nearly along the light cone, because of the light pion mass. The  $\omega_0$  and  $\omega_r$  fields are shown in Figures 2 and 3. Recall that  $\omega_r$  is initially zero. We see that both emerge more slowly and with more complex behavior than the pion field, due to the large omega mass. The following figures show the energy density. First we see (in Figure 4) the total energy density then in Figures 5 and

6 the energy density in the pion field and in the r-component of the omega field. That is the part of the omega that corresponds to radiation. In all these plots we show the energy density multiplied by  $4\pi r^2$ , that is the total energy at any time is obtained from them by integration over  $dr$  only. For the total, this integral must yield 2 GeV at every time, and we use this test to verify the stability of our calculation. Without the factor of  $r^2$ , the energy density would be seen to decrease rapidly with  $r$ , reflecting the corresponding decrease in the fields. This decrease is most dramatically seen in Figure 7 where we plot the interaction energy density. We see that past the first few time steps, the interaction energy disappears, signifying the decoupling of the fields. Beyond this point not only do the fields decouple, but, to a very good approximation, they obey a linear wave equation. This region is the radiation zone, and there we can use Eqs. (19) and (20) to obtain the pion and omega momentum distributions. In Figures 8 and 9 we plot the reduced momentum distribution densities which are defined by

$$\rho_f(\bar{k}) = 4\pi m k^2 |f(k)|^2 \quad (21)$$

and

$$\rho_g(\bar{k}) = 4\pi m k^2 |g(k)|^2 \quad (22)$$

as functions of scaled dimensionless wave number  $\bar{k} = k/m$ . We see that the pion distribution is quite sharply peaked, roughly at a wave number corresponding to the size of the initial distribution. The omega distribution is much broader, largely because that field propagates with a mass that is large compared with other scales in the problem. The momentum amplitudes,  $f(k)$  and  $g(k)$  are the basis for our coherent state quantization.

It is amusing to compare the dynamical calculation reported above with our previous phenomenological description of annihilation into pions only, [1], [2]. We can repeat the dynamical calculation done above using the same initial form,  $F(r, t = 0)$ , for the baryonic configuration, but this time having no omega field and using the standard Skyrme lagrangian [12]. This is the appropriate dynamics for our previous, pions only, picture. Once again we propagate the pion field into the radiation zone and calculate the momentum density,  $f(k)$ . The corresponding reduced distribution densities are shown in Figure 10, where it is compared with the phenomenological form used in [1], [2]. We see that the two are remarkably alike. Their overall scale agrees because they were fit to the same total energy, and they both come from an initial distribution of about the same size, but they were arrived at very differently and their agreement is certainly both satisfying and surprising.

### III. COHERENT STATE CALCULATION

We construct quantum coherent states for the  $\omega$  and  $\pi$  mesons from the classical fields obtained in the previous section using the same methods we used before [1], [2]. Recall that we wish to construct coherent states that have fixed four-momentum and isospin. To do this we define a pion field operator that creates pions at the space-time position  $x$  and pointing in the isospin direction  $\hat{T}$  by

$$F(x) = \int d^3k f(\vec{k}) \vec{a}_{\vec{k}}^\dagger \cdot \hat{T} e^{-ik \cdot x}, \quad (23)$$

where in the exponent under the integral, the energy component of  $k$  is taken as the on-shell value  $k_0 = \sqrt{k^2 + m_\pi^2}$ . The corresponding omega field operator is given by

$$G(x) = \int d^3k' g(\vec{k}') b_{\vec{k}'}^\dagger e^{-ik' \cdot x}. \quad (24)$$

with appropriate definition of the fourth component of  $k'$ . In these forms,  $\vec{a}_{\vec{k}}^\dagger$  creates an isovector pion with momentum  $\vec{k}$  and  $b_{\vec{k}'}^\dagger$  creates an isoscalar omega with momentum  $\vec{k}'$ .

To impose both energy-momentum and isospin conservations, we take the quantum coherent state of the pion and omega system in the radiation region to be given by

$$|I, I_z, K\rangle = \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}}{\sqrt{4\pi}} e^{iK \cdot x} Y_{II_z}(\hat{T}) (e^{F(x)+G(x)} - 1 - F(x) - G(x)) |0\rangle. \quad (25)$$

As we explain in [1], we subtract the “one” as well as the one particle (pion or omega) states since they are not permitted by energy-momentum conservation and make the calculation numerically unstable. The states defined in (25) are not normalized, but they are orthogonal. We have

$$\langle I, I_z, K | I', I'_z, K' \rangle = \delta^4(K - K') \delta_{II'} \delta_{I_z I'_z} \mathcal{I}(K) \quad (26)$$

The normalization factor is given by

$$\mathcal{I}(K) = \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T} d\hat{T}'}{4\pi} Y_{II_z}^*(\hat{T}) Y_{II_z}(\hat{T}') e^{iK \cdot x} (e^{\rho_\pi(x)\hat{T} \cdot \hat{T}' + \rho_\omega(x)} - 1 - \rho_\pi(x)\hat{T} \cdot \hat{T}' - \rho_\omega(x)) \quad (27)$$

where

$$\rho_\pi(x) = \int d^3p |f(\vec{p})|^2 e^{-ip \cdot x} \quad (28)$$

and

$$\rho_\omega(x) = \int d^3p |g(\vec{p})|^2 e^{-ip \cdot x} \quad (29)$$

The normalization integral is difficult to calculate numerically, even after the subtraction. Hence we use the expansion method we developed before [2], generalized to the case of two meson types. We thus get

$$\mathcal{I}(K) = \sum_{N_\pi + N_\omega \geq 2} \frac{I(K, N_\pi, N_\omega)}{N_\pi! N_\omega!} F(N_\pi, I). \quad (30)$$

The probability for finding  $N_\pi$  pions and  $N_\omega$  omega is given by

$$p(N_\pi, N_\omega) = \frac{1}{\mathcal{I}(K)} \frac{I(K, N_\pi, N_\omega)}{N_\pi! N_\omega!} F(N_\pi, I) \quad (31)$$

where

$$I(K, N_\pi, N_\omega) = \int \delta^4(K - \sum_{i=1}^{N_\pi} p_i - \sum_{j=1}^{N_\omega} q_j) \prod_{i=1}^{N_\pi} d^3p_i |f(\vec{p}_i)|^2 \prod_{j=1}^{N_\omega} d^3q_j |g(\vec{q}_j)|^2 \quad (32)$$

and

$$F(N_\pi, I) = \int \frac{d\hat{T}d\hat{T}'}{4\pi} Y_{II_z}^*(\hat{T}) Y_{II_z}(\hat{T}') (\hat{T} \cdot \hat{T}')^{N_\pi} \\ = \begin{cases} 0 & I > N_\pi \text{ and } I - N_\pi \text{ is odd} \\ \frac{N_\pi!}{(N_\pi - I)!!(I + N_\pi + 1)!!} & I \leq N_\pi \text{ and } I - N_\pi \text{ is even.} \end{cases} \quad (33)$$

If either  $N_\pi$  or  $N_\omega$  equal zero, the corresponding terms will not appear in Eq.(32).

The mean number of  $\pi$ 's of isospin type  $\mu$  in the state is given by

$$N_{\pi\mu} = \frac{1}{\mathcal{I}} \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}d\hat{T}'}{4\pi} Y_{II_z}^*(\hat{T}) Y_{II_z}(\hat{T}') \hat{T}_\mu \hat{T}'_\mu e^{iK \cdot x} \rho_\pi(x) (e^{\rho_\pi(x) \hat{T} \cdot \hat{T}' + \rho_\omega(x)} - 1), \quad (34)$$

and the mean number of  $\omega$ 's by

$$N_\omega = \frac{1}{\mathcal{I}} \int \frac{d^4x}{(2\pi)^4} \frac{d\hat{T}d\hat{T}'}{4\pi} Y_{II_z}^*(\hat{T}) Y_{II_z}(\hat{T}') e^{iK \cdot x} \rho_\omega(x) (e^{\rho_\pi(x) \hat{T} \cdot \hat{T}' + \rho_\omega(x)} - 1). \quad (35)$$

These can again be calculated using the expansion method. It is clear that correlations can be calculated as before [2], but we will not do so here.

#### IV. RESULTS

Our calculational plan is now clear. We begin with a spherically symmetric “blob” of Skyrmionic matter of size fixed by the  $\pi$ -meson mass and amplitude fixed by a total energy of 2 GeV, but with baryon number zero. We use Skyrme dynamical equations modified to include the omega field to propagate the classical pion and omega fields outward into the radiation zone. These classical radiation fields are then used to construct quantum coherent states for the  $\pi$ -mesons and for the  $\omega$ -mesons, and the coherent states are projected onto states of good isospin and four-momentum. We use these states to find mean meson numbers. Note that except for the parameters in the Skyrme model, and these are fixed by nucleon physics, there are no free parameters in our picture.

We first calculate mean meson numbers from the purely classical fields. This is completely equivalent to calculating the mean numbers in the coherent state without the isospin and four-momentum projections. We find

$$N_\pi = 4.4$$

$$N_\omega = 0.86$$

$$N_{\text{total}} = 7.0$$

where  $N_{\text{total}}$  is the total number of pions, coming from both direct pions and from omega decay. It is given by  $N_{\text{total}} = N_\pi + 3N_\omega$ . Our case corresponds to a percentage of secondary pions of 37%. Note that the mean number of omega mesons is quite small. It is usual to say that coherent state methods are best applied in the limit of a large number of quanta, and 0.86 is certainly not a large number. Nevertheless we use coherent states methods for this case both to remain parallel with the treatment of the pions, and because we have little

idea of what else to do. An after-the-fact justification comes from the close agreement with data.

We now turn to a calculation of the number of pions and omega meson from annihilation with both isospin and four-momentum projections included as in (34) and (35). In the table we show the number of pions (now separated by charge type) and number of omegas for each of the isospin channels that can be reached in nucleon-antinucleon annihilation at rest. We also show the total number of pions of each charge type as well as the total number of all types. These are calculated combining the direct pions with those from omega decay using the decay mode  $\omega \rightarrow \pi^+ + \pi^0 + \pi^-$ . We also give the pion variance for each channel. We see, as we expect from its large mass, that the mean number of omega mesons is decreased by the imposition of energy and momentum conservation. The mean number of pions, the variance, and the fraction by charge type is quite close to what is observed [8]. The percentage of secondary pions is 23% for all three isospin channels. This too is roughly what is seen when allowance is made for pions from rho mesons, that we have not yet included. It may be argued that since the mean number of pions comes out about right, there is no room for pions from rho mesons. That argument is incorrect, because it neglects the important effect of energy conservation. Note that including omega mesons has not substantially raised the mean number of pions from what we obtain in our earlier, pions only, calculation [1], [2]. Because the total energy is fixed, when we add degrees of freedom, the energy is redistributed, less goes into direct pions and more into indirect, but the total remains about the same. We expect this will be true when rho mesons are included, but that then the number of secondary pions will be close to the experimental value of 40%.

The pion averages are interesting, but the pion number spectrum is a more stringent test of the formalism. We are interested in the probability,  $P_n$ , of finding exactly  $n$  pions in annihilation at rest, from states of fixed isospin and four-momentum. This can be calculated from the joint probability of finding  $N_\pi$  pions, and  $N_\omega$  omega mesons,  $p(N_\pi, N_\omega)$  using the relation

$$P_n = \sum_{n=N_\pi+3N_\omega} p(N_\pi, N_\omega). \quad (36)$$

The joint probability,  $p(N_\pi, N_\omega)$ , is given by Eq.(31). This is the generalization to two meson types of the method used for pions only in [2]. In Figures 11, 12 and 13, we show  $P_n$  as a function of  $n$  for each of the three isospin cases. The solid squares are the results of our calculation, the open circles are a normalized gaussian distribution fixed to the same average and variance as our calculation. We see, as we saw in our previous work [1], [2], that with four-momentum conservation imposed, our calculation is indistinguishable from a gaussian distribution, even though our reaction mechanism is very far from a statistical one. The pion number distributions in the three figures look very much like the empirical ones [13]. In our previous, pions only, calculation [2], we found that  $I = 0$  states could go only into even numbers of pions and  $I = 1$  states into odd numbers. No such effect is seen in the data. Note that by including the omega meson we have smoothly removed any vestige of this even-odd effect. All of this gives further support to the quantized classical wave picture of annihilation.

## V. CONCLUSION

We have shown that a classical treatment of nucleon-antinucleon annihilation at rest based on Skyrme dynamics extended to include the omega meson and then quantized using coherent states gives an excellent account of the pion spectra seen in annihilation. This treatment also removes an odd-even artifact of the pions only treatment we presented before [1], [2]. The treatment presented here also is more firmly based in the classical dynamics and hence more rooted in QCD. In fact our treatment now has no free parameters, and yet fits the major trends of the data very well. Its major failing is that we find only 23% of annihilations going via vector mesons, while empirically that number is closer to 40%. This missing fraction is presumably due to annihilations into rho mesons, which are not yet in our picture. We argue that including them will boost the fraction of annihilations into vector mesons while energy-momentum conservation will prevent that boost from spoiling agreement with pion number distributions.

Our approach to annihilation is part of a widening family of approaches to strong interaction physics problems that fall squarely in the domain of non-perturbative QCD, but that involve fairly large energy release. These problems are very difficult to solve in full QCD. The alternative is to solve them first in classical QCD (CCD). There the dynamics is difficult but often tractable. In our case it is the Skyrme model modified to include omega mesons. Once the classical fields have been obtained, they can be quantized in the radiation zone using the method of coherent states and some of the important quantum numbers can even be imposed. These quantized states can then be compared with experiment. In our case we find good agreement, as far as we have gone, and are encouraged to go further. In particular we will study pion correlations, annihilation in flight, the effect of the rho meson and related phenomena. It should be noted that one of the benefits of the classical dynamics-coherent state approach is that all final channels are treated together. Further afield this direction, using classical QCD first and quantizing later, is being applied to disoriented chiral condensates, Centauro events [14], [15], [16], [17], [18], [19], and much else. It may have other uses in high energy heavy ions collisions and in other problems of coherent hadronization.

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## Figure Captions

Figure 1. Pion field configuration  $F$  as a function of  $r$  and  $t$ .

Figure 2.  $\omega_0$  (in units of omega mass  $m$ ) field as a function of  $r$  and  $t$ .

Figure 3.  $\omega_r$  (in units of omega mass  $m$ ) field as a function of  $r$  and  $t$ .

Figure 4. Total energy density multiplied by  $4\pi r^2$  (in units of MeV/fm) as a function of  $r$  and  $t$ .

Figure 5. Pion energy density multiplied by  $4\pi r^2$  (in units of MeV/fm) as a function of  $r$  and  $t$ .

Figure 6. Energy density associated with  $\omega_r$  multiplied by  $4\pi r^2$  (in units of MeV/fm) as a function of  $r$  and  $t$ .

Figure 7. Interaction energy density multiplied by  $4\pi r^2$  (in units of MeV/fm) as a function of  $r$  and  $t$ .

Figure 8. Pion momentum distribution. The horizontal axis is the scaled momentum in units of inverse omega mass. The vertical axis is the dimensionless reduced pion distribution function. The area under the curve gives the total direct pion number  $N_\pi$ .

Figure 9. Omega momentum distribution. The horizontal axis is the scaled momentum in units of inverse omega mass. The vertical axis is the dimensionless reduced omega distribution function. The area under the curve gives the total omega number  $N_\omega$ .

Figure 10. Pion momentum distribution. Solid line: Skyrme calculation; Dot-Dashed Line: Phenomenological pion momentum distribution used in [1,2]. The horizontal axis is the scaled momentum in units of inverse omega mass. The vertical axis is the dimensionless reduced pion distribution function. The area under the curve gives the total pion number  $n$ .

Figure 11. Pion number distribution in  $I = 0, I_z = 0$  channel. Solid squares are given by the distribution  $P_n$ . Circles are the gaussian distribution with the same mean and variance.

Figure 12. Pion number distribution in  $I = 1, I_z = 0$  channel. Solid squares are given by the distribution  $P_n$ . Circles are the gaussian distribution with the same mean and variance.

Figure 13. Pion number distribution in  $I = 1, I_z = 1$  channel. Solid squares are given by the distribution  $P_n$ . Circles are the gaussian distribution with the same mean and variance.

## TABLES

TABLE I. pion number distribution.  $N_\pi$  is the total direct pion number and  $N_\omega$  is the total omega number.  $n_0$  is the total  $\pi^0$  number including those from omega decay. Similarly  $n_+$  and  $n_-$  are the total  $\pi^+$  and  $\pi^-$  numbers including those from omega decay.  $n$  is the total pion number and  $\sigma$  is the standard deviation for  $n$  calculated from the distribution function  $P_n$ .

Channel	$N_\pi$	$N_\omega$	$n_0$	$n_+$	$n_-$	$n$	$\sigma$
$I = 0 \quad I_z = 0$	4.94	0.60	2.25	2.25	2.25	6.75	0.79
$I = 1 \quad I_z = 0$	5.17	0.48	3.99	1.31	1.31	6.61	0.84
$I = 1 \quad I_z = 1$	5.20	0.48	1.32	3.16	2.16	6.64	0.84

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